Rossmoyne Senior High School

Semester One Examination, 2019

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 3

Section Two: Calculator-assumed

SOLUTIONS

Student number: In figures

In words: ______________________________

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Circle your teacher's name: Ms Chua Ms Robinson Mr Tan

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	99	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (98 Marks)

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 (6 marks)

Consider the following system of equations, where a and b are constants.

$$\begin{aligned} x + 2y + 2z &= 5 \\ 3x + 2y + 4z &= 9 \\ 3x + ay + 2z &= b \end{aligned}$$

For each of the following cases, determine the number of solutions that exist for the system and give a geometric description of the situation.

(a) $a = 1,\ b = 3.$ (3 marks)

Solution
The system has 1 solution. The system represents three planes that intersect at the point $(-1, 0, 3)$. $\begin{cases} x+2y+2z=5 \\ 3x+2y+4z=9 \\ 3x+a\times y+2z=b \end{cases}\Big\vert_{x,y,z}$ $\left\{x=\frac{-(a-2\cdot b+8)}{a+2},\ y=\frac{b-3}{a+2},\ z=\frac{3\cdot a-2\cdot b+12}{a+2}\right\}$ ans | a=1 | b=3 $\{x=-1, y=0, z=3\}$
Specific behaviours
✓ number of solutions ✓ interpretation ✓ interpretation includes point of intersection

(b) $a = -2,\ b = 3.$ (3 marks)

Solution
The system has an infinite number of solutions. The system must represent three planes intersecting in a straight line, as no two planes are parallel. ans | a=−2 | b=3 Undefined
Specific behaviours
✓ number of solutions ✓ interpretation ✓ interpretation refers to non-parallel planes

Question 9 **(7 marks)**

(a) Determine the values of the real constant a and the real constant b given that $z - 4 + 2i$ is a factor of $z^3 + az + b$.

(4 marks)

Solution
Let $z = 4 - 2i$, then $z^3 = 16 - 88i$ Hence $16 - 88i + 4a - 2ai + b = 0$ Re parts: $16 + 4a + b = 0$ Im parts: $-88 - 2a = 0$ Hence $a = -44, b = 160$
Specific behaviours
✓ identifies root and substitutes ✓ equates real and imaginary parts to zero ✓ solves for a ✓ correct values

(b) (i) Clearly show that $2 + i$ is a root of the equation $z^3 - 7z^2 + 17z - 15 = 0$. (2 marks)

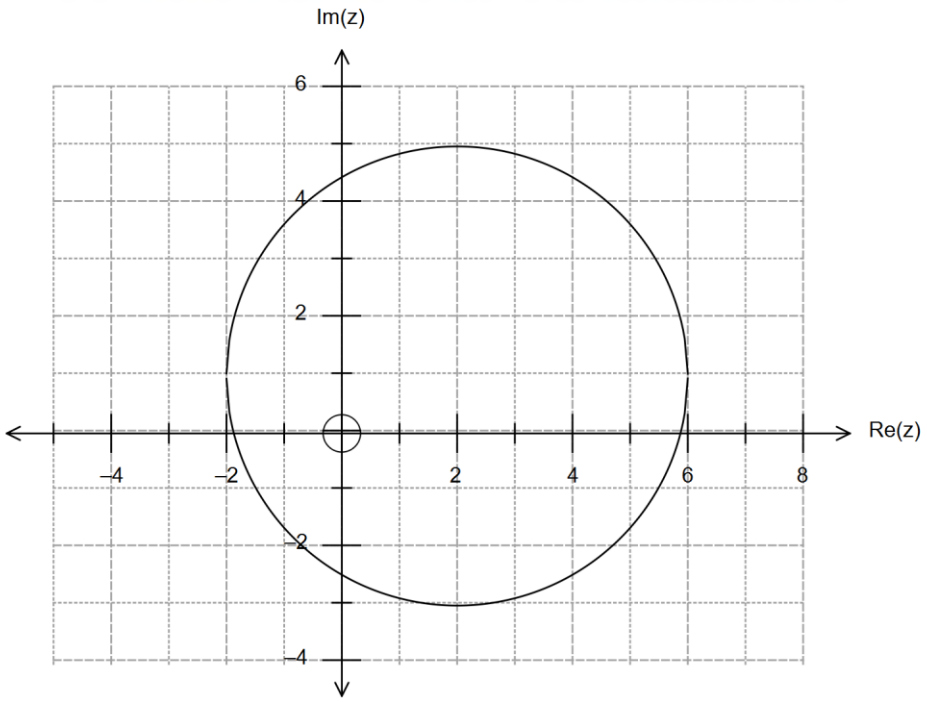
Solution
$z = 2 + i, 17z = 34 + 17i, 7z^2 = 21 + 28i, z^3 = 2 + 11i$ $z^3 - 7z^2 + 17z - 15 = 2 + 11i - 21 - 28i + 34 + 17i - 15$ $= 36 - 36 + 28i - 28i$ $= 0$
Specific behaviours
✓ shows expanded term for z^3 ✓ fully expands all terms and sums to zero

(ii) State all three solutions of $z^3 - 7z^2 + 17z - 15 = 0$. (1 mark)

Solution
$z = 3, 2 + i, 2 - i$
Specific behaviours
✓ correct solutions

Question 10 **(6 marks)**

The diagram below shows the region represented by $|z-2-i|=4$



(a) Determine the minimum and maximum value of $|z|$. (2 marks)

Solution
$\|z\|_{max}$ and $\|z\|_{min}$ occur along the line connecting the centre of the circle with the origin.
$\left\|\begin{matrix}2\\1\end{matrix}\right\| = \sqrt{5}$
$\therefore \|z\|_{max} = 4+\sqrt{5}$ and $\|z\|_{min} = 4-\sqrt{5}$
Specific behaviours
✓ $\|z\|_{min}$ ✓ $\|z\|_{max}$

(b) Determine the value(s) of $arg(z)$ for when $Re(z)=4$. (4 marks)

Solution
$(x-2)^2+(y-1)^2=16$ and $Re(z)=x=4$
$\therefore 4+(y-1)^2=16 \quad \rightarrow \quad y=1\pm\sqrt{12}$
$\therefore \arg(z)=\tan^{-1}\left(\dfrac{1\pm\sqrt{12}}{4}\right)=0.8402^R \ or \ -0.5521^R$
Specific behaviours
✓✓ $4+(y-1)^2=16 \quad \rightarrow \quad y=1\pm\sqrt{12}$ ✓✓ $\arg(z)=\tan^{-1}\left(\frac{1\pm\sqrt{12}}{4}\right)=0.8402^R \ or \ -0.5521^R$

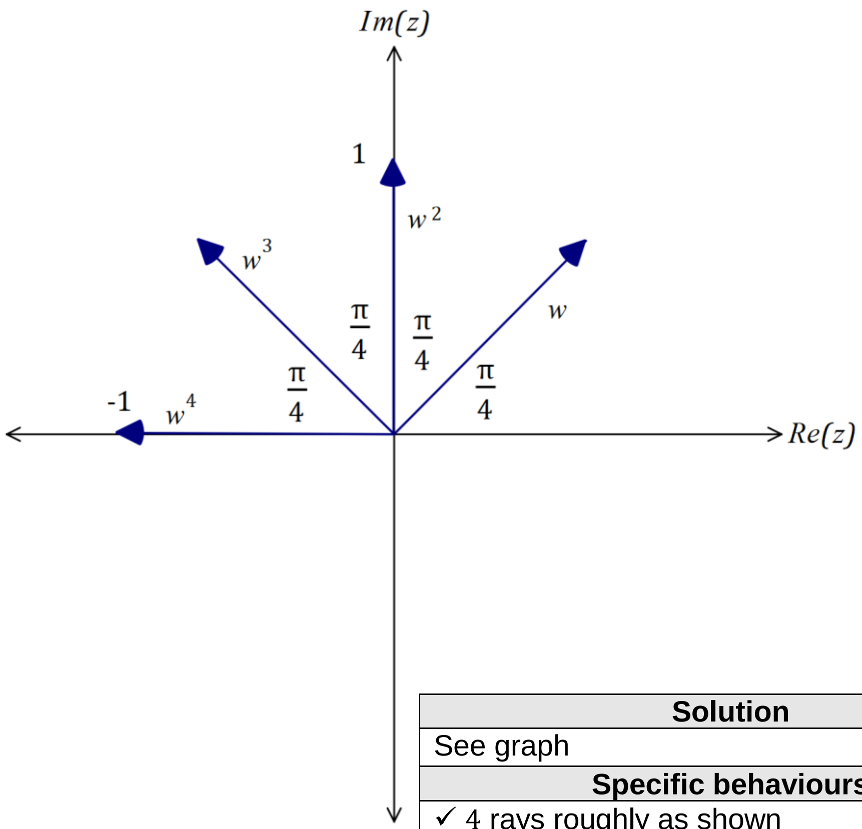
Question 11 **(9 marks)**

Let $w = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$.

(a) Express w, w^2, w^3 and w^4 in the form $r\,\mathrm{cis}\,\theta$, $-\pi < \theta \le \pi$. (2 marks)

Solution
$w = \mathrm{cis}\left(\frac{\pi}{4}\right), w^2 = \mathrm{cis}\left(\frac{\pi}{2}\right), w^3 = \mathrm{cis}\left(\frac{3\pi}{4}\right), w^4 = \mathrm{cis}(\pi)$
Specific behaviours
✓ w correct ✓ all correct

(b) Sketch w, w^2, w^3 and w^4 as vectors on the Argand diagram below. (2 marks)



Solution
See graph
Specific behaviours
✓ 4 rays roughly as shown ✓ indicates scale and angle between rays

(c) Describe the transformation in the complex plane of any point z when it is multiplied by w. (2 marks)

Solution
Rotation (anti clockwise) about origin of $\frac{\pi}{4}$
Specific behaviours
✓ at least one element of transformation ✓ all three elements of transformation

(d) Simplify

(i) $w + w^3 + w^5 + w^7$. (1 mark)

Solution
0
Specific behaviours
✓ correct value

(ii) $w + w^3 + w^5 + \cdots + w^{2017} + w^{2019}$. (2 marks)

Solution
$w + w^3 + w^5 + \cdots + w^{2015} = 0$ $w^{2017} + w^{2019} = w + w^3 = \sqrt{2}i$
Specific behaviours
✓ correct sum for $w + \cdots + w^{2015}$ ✓ correct value

Question 12 **(9 marks)**

The position vector of a small body, in metres, is $\mathbf{r}(t) = (2 + 4\sin(t))\mathbf{i} + (2\cos(2t) - 1)\mathbf{j}$ where t is the time in seconds since motion began.

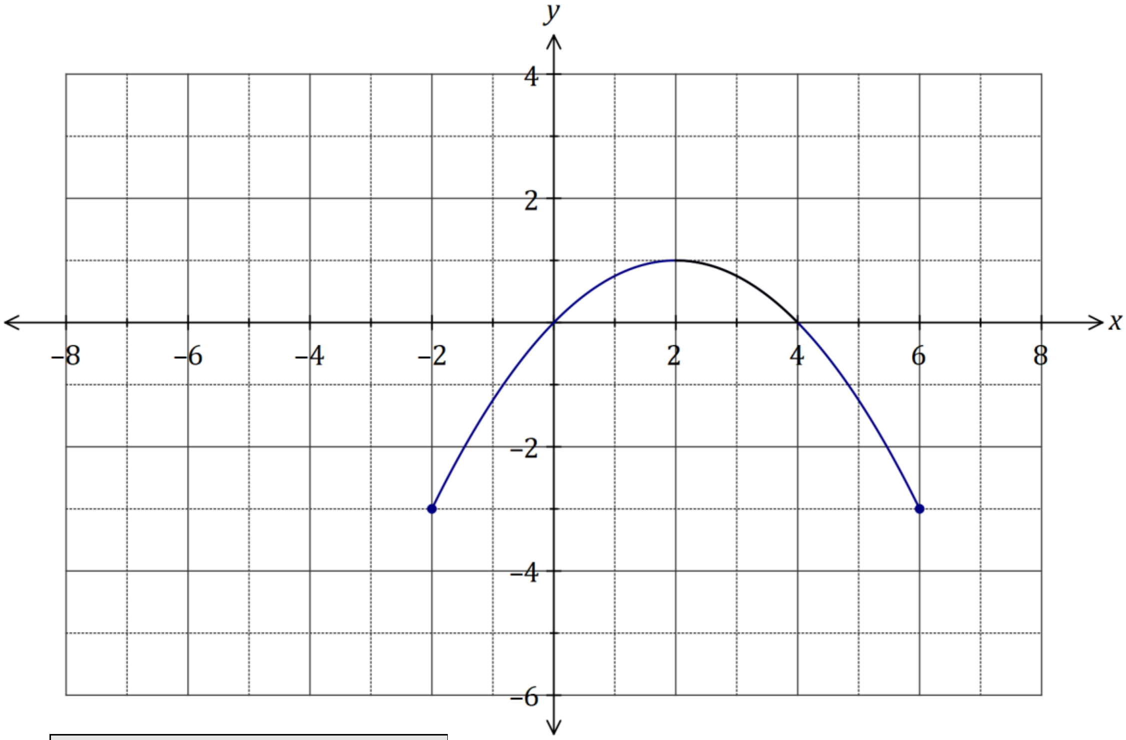
(a) Show that the body is stationary when $t = \frac{\pi}{2}$ and state its position at this time. (3 marks)

Solution
$\mathbf{v}(t) = 4\cos(t)\,\mathbf{i} - 4\sin(2t)\mathbf{j}$ $\mathbf{v}\left(\frac{\pi}{2}\right) = 4\cos\left(\frac{\pi}{2}\right)\mathbf{i} - 4\sin(\pi)\,\mathbf{j} = \mathbf{0}$ $\mathbf{r}\left(\frac{\pi}{2}\right) = \left(2 + 4\sin\left(\frac{\pi}{2}\right)\right)\mathbf{i} + (2\cos(\pi) - 1)\mathbf{j} = 6\mathbf{i} - 3\mathbf{j}$
Specific behaviours
✓ expression for velocity ✓ substitutes time and obtains zero vector ✓ states position

(b) Derive the Cartesian equation of the path of the body. (4 marks)

Solution
$y = 2\cos(2t) - 1 = 2(1 - 2\sin^2(t) - 1) = 1 - 4\sin^2(t)$ $x = 2 + 4\sin(t) \Rightarrow \sin^2(t) = \dfrac{(x-2)^2}{16}$ $y = 1 - \dfrac{(x-2)^2}{4}$ where $-2 \le x \le 6$
Specific behaviours
✓ expression for y in terms of $\sin^2(t)$ ✓ expression for $\sin^2(t)$ in terms of x ✓ Cartesian equation (accept different forms) ✓ restricts domain or range

(c) Complete the following plot to show the path taken by the body. (2 marks)



Solution
See graph
Specific behaviours
✓ parabola ✓ correct domain

Question 13 **(7 marks)**

(a) Solve the equation $z^5 + 32 = 0$, writing your solutions in polar form $r \operatorname{cis} \theta$. (4 marks)

Solution
$z^5 = -32$ $= 2^5 \operatorname{cis} \pi$ $z_n = 2 \operatorname{cis}\left(\pi - \frac{2n\pi}{5}\right), n = 0,1,2,3,4$ $z_0 = 2\operatorname{cis}(\pi), z_1 = 2\operatorname{cis}\left(\frac{3\pi}{5}\right), z_2 = 2\operatorname{cis}\left(\frac{\pi}{5}\right), z_3 = 2\operatorname{cis}\left(\frac{-\pi}{5}\right), z_4 = 2\operatorname{cis}\left(\frac{-3\pi}{5}\right)$
Specific behaviours
✓ expresses in polar form ✓ states general solution ✓ states one correct solution in polar form ✓ states all correct solutions in polar form

(b) Use your answers from (a) to show that $\cos\left(\frac{\pi}{5}\right) + \cos\left(\frac{3\pi}{5}\right) = \frac{1}{2}$. (3 marks)

Solution
Since $z_0 + z_1 + z_2 + z_3 + z_4 = 0$ then $\operatorname{Re}(z_0 + z_1 + z_2 + z_3 + z_4) = 0$ $2\cos(\pi) + 2\cos\left(\frac{3\pi}{5}\right) + 2\cos\left(\frac{\pi}{5}\right) + 2\cos\left(-\frac{\pi}{5}\right) + 2\cos\left(-\frac{3\pi}{5}\right) = 0$ But $\cos(-\theta) = \cos\theta$ and $\cos\pi = -1$ Hence $2\cos\left(\frac{\pi}{5}\right) + 2\cos\left(\frac{3\pi}{5}\right) + 2\cos\left(\frac{\pi}{5}\right) + 2\cos\left(\frac{3\pi}{5}\right) = 2$ $\cos\left(\frac{\pi}{5}\right) + \cos\left(\frac{3\pi}{5}\right) = \frac{2}{4} = \frac{1}{2}$
Specific behaviours
✓ indicates that sum of roots is zero ✓ equates real part of sum of roots to zero ✓ states $\cos(-\theta) = \cos\theta$ and $\cos\pi = -1$ and simplifies

Question 14 **(8 marks)**

The position vectors of two particles at time t are given below, where a is a constant.

$$\mathbf{r}_A = 21\mathbf{i} - 11\mathbf{j} + 14\mathbf{k} + t(-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \text{ and } \mathbf{r}_B = 5\mathbf{i} - 3\mathbf{j} + a\mathbf{k} + t(5\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

The paths of the particles cross at P but the particles do not meet.

(a) Determine the value of the constant a and the position vector of P. (5 marks)

Solution

$$\mathbf{r}_A = \begin{pmatrix} 21-2t \\ -11+2t \\ 14-t \end{pmatrix}, \mathbf{r}_B = \begin{pmatrix} 5+5s \\ -3-s \\ a-2s \end{pmatrix}$$

Hence $21 - 2t = 5 + 5s$ and $-11 + 2t = -3 - s \Rightarrow t = 3, s = 2$

Using $\mathbf{k}$ coefficient: $14 - 3 = a - 2(2) \Rightarrow a = 15$

$$\mathbf{r}_A(3) \Rightarrow \overrightarrow{OP} = \begin{pmatrix} 15 \\ -5 \\ 11 \end{pmatrix}$$

Specific behaviours

✓ replaces one t with another variable (e.g. s)
✓ uses $\mathbf{i}$ and $\mathbf{j}$ components to write pair of equations
✓ solves equations for t and s
✓ substitutes into $\mathbf{k}$ components and determines a
✓ uses t or s to find P

ClassPad Solution

Method 1

1

$\begin{cases} 21-2t=5+5s \\ -11+2t=-3-s \\ 14-t=a-2s \end{cases}\Bigg|_{t,s,a}$

{t=3, s=2, a=15}

Method 2

2

solve$\left(\begin{bmatrix} 21 \\ -11 \\ 14 \end{bmatrix}+t\times\begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix}=\begin{bmatrix} 5 \\ -3 \\ a \end{bmatrix}+s\times\begin{bmatrix} 5 \\ -1 \\ -2 \end{bmatrix}, \{t, s,\right.$

{t=3, s=2, a=15}

$\begin{bmatrix} 21 \\ -11 \\ 14 \end{bmatrix}+t\times\begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix} \mid t=3$

$\begin{bmatrix} 15 \\ -5 \\ 11 \end{bmatrix}$

crossP$\left(\begin{bmatrix} 5 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix}\right)$

$\begin{bmatrix} 5 \\ 9 \\ 8 \end{bmatrix}$

dotP$\left(\begin{bmatrix} 5 \\ 9 \\ 8 \end{bmatrix}, \begin{bmatrix} 21 \\ -11 \\ 14 \end{bmatrix}\right)$

118

(b) Show that the point $(4, 10, 1)$ lies in the plane containing the two lines. (3 marks)

Solution

$$(5\mathbf{i} - \mathbf{j} - 2\mathbf{k}) \times (-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = 5\mathbf{i} + 9\mathbf{j} + 8\mathbf{k}$$

$$(5\mathbf{i} + 9\mathbf{j} + 8\mathbf{k}) \cdot (21\mathbf{i} - 11\mathbf{j} + 14\mathbf{k}) = 118$$

Equation of plane is $\mathbf{r} \cdot (5\mathbf{i} + 9\mathbf{j} + 8\mathbf{k}) = 118$

$$(4\mathbf{i} + 10\mathbf{j} + \mathbf{k}) \cdot (5\mathbf{i} + 9\mathbf{j} + 8\mathbf{k}) = 20 + 90 + 8 = 118$$

Hence point lies in plane.

Specific behaviours

✓ calculates normal to plane
✓ calculates constant and writes equation of plane
✓ substitutes point, showing equation satisfied

Question 15 **(8 marks)**

Sphere S has diameter PQ, where P and Q have coordinates $(2, -3, 1)$ and $(-4, 7, 5)$ respectively.

(a) Determine the vector equation of the sphere. (3 marks)

Solution
$\overrightarrow{OC} = \frac{1}{2}(P + Q) = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$ $r = \left\| \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \right\| = \left\| \begin{pmatrix} -3 \\ 5 \\ 2 \end{pmatrix} \right\| = \sqrt{38}$ $\left\| \mathbf{r} - \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \right\| = \sqrt{38}$
Specific behaviours
✓ indicates position of centre   ✓ indicates radius   ✓ correct vector equation

(b) Show that the point $(1, -1, 2)$ lies inside the sphere. (2 marks)

Solution
$\left\| \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \right\| = \left\| \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} \right\| = \sqrt{14}$ Since $\sqrt{14} < \sqrt{38}$, point lies inside sphere.
Specific behaviours
✓ calculates distance   ✓ explains result

(c) Show that the line with equation $\mathbf{r} = \begin{pmatrix} 1 \\ -3 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is tangential to the sphere. (3 marks)

Solution
$\left\| \begin{pmatrix} 1 + \lambda \\ -3 + \lambda \\ -3 + \lambda \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \right\| = \sqrt{38}$ $(2 + \lambda)^2 + (-5 + \lambda)^2 + (-6 + \lambda)^2 = 38 \Rightarrow \lambda = 3$ As λ has a unique value, the line only intersects sphere at one point and so it must be a tangent.
Specific behaviours
✓ substitutes line equation into sphere equation   ✓ solves for λ ✓ explains result

Question 16 **(9 marks)**

Let $f(x) = \sqrt{x-1}$, $g(x) = \dfrac{3}{x}$ and $h(x) = f \circ g(x)$.

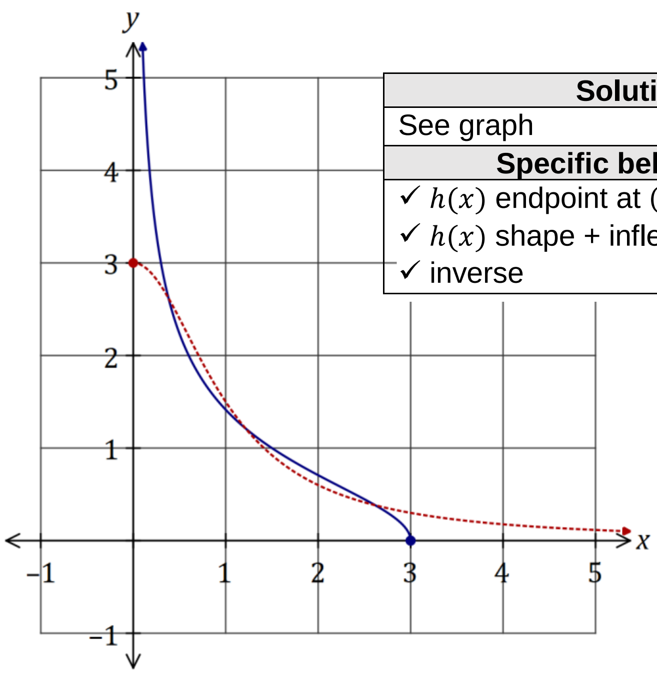
(a) Determine an expression for $h(x)$ and show that the domain of $h(x)$ is $0 < x \le 3$. (3 marks)

Solution
$$h(x) = \sqrt{\frac{3}{x} - 1}$$ D_h: (i) require $x > 0$ so that $\frac{3}{x} - 1 > 0$ and (ii) $\frac{3}{x} \ge 1 \Rightarrow x \le 3$ Hence $D_h: \{x \in \mathbb{R}: 0 < x \le 3\}$
Specific behaviours
✓ $h(x)$ ✓ explains why $x > 0$ ✓ explains why $x \le 3$

(b) Determine an expression for $h^{-1}(x)$, the inverse of $h(x)$. (1 mark)

Solution
$$h^{-1}(x) = \frac{3}{x^2+1} \quad \text{(CAS)}$$
Specific behaviours
✓ correct expression

(c) Sketch the graphs of $y = h(x)$ and $y = h^{-1}(x)$ on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ $h(x)$ endpoint at $(3,0)$ ✓ $h(x)$ shape + inflection $x \approx 2$ ✓ inverse

(d) Solve $h(x) = h^{-1}(x)$, correct to 0.01 where necessary. (2 mark)

Solution
$x \approx 0.38, \quad x \approx 1.21, \quad x \approx 2.62 \quad \text{(CAS)}$
Specific behaviours
✓✓ correct solutions (-1 per error)   -1 if not correct to 0.01 at least 2 of 3 ans

Question 17 **(9 marks)**

A pole and a wall stand vertically on horizontal ground. A small projectile is launched from the pole at a height of 3.16 m above the ground and sometime later hits the wall at a height of 1.79 m above the ground. The projectile has an initial velocity of 32 ms^{-1} at an angle of $36°$ above the horizontal.

Any effects of air resistance and wind can be ignored. Let $\mathbf{i}$ and $\mathbf{j}$ be unit vectors in the horizontal and vertical (upward) directions and the foot of the pole be at $(0, 0)$.

The acceleration acting on the projectile is given by $\mathbf{a}(t) = -9.8\mathbf{j}$ ms^{-2}.

(a) Use the information above to derive vector equations for the velocity $\mathbf{v}(t)$ and displacement $\mathbf{r}(t)$ of the projectile at any time t. (3 marks)

Solution
$\mathbf{v}(t) = (32\cos 36°)\mathbf{i} + (32\sin 36° - 9.8t)\mathbf{j}$ $\mathbf{r}(t) = (32t\cos 36°)\mathbf{i} + (3.16 + 32t\sin 36° - 4.9t^2)\mathbf{j}$
Specific behaviours
✓ integrates correctly twice ✓ correct expression for $\mathbf{v}(t)$ ✓ correct expression for $\mathbf{r}(t)$

(b) Determine

(i) the time that the projectile takes to travel between the pole and the wall. (2 marks)

Solution
$3.16 + 32t\sin 36° - 4.9t^2 = 1.79$ $t = 3.91$ s
Specific behaviours
✓ equates $\mathbf{j}$ coefficient of displacement to height ✓ solves for time

(ii) the speed of the projectile at the instant it hits the wall. (2 marks)

Solution
$\mathbf{v}(3.91) = 25.89\mathbf{i} - 19.51\mathbf{j}$ $\lvert\mathbf{v}(3.91)\rvert = 32.4\ \mathrm{ms}^{-1}$
Specific behaviours
✓ indicates velocity at correct time ✓ correct speed (including units).

(iii) the distance travelled by the projectile between the pole and the wall. (2 marks)

Solution
$l = \int_0^{3.91} \lvert\mathbf{v}(t)\rvert\, dt$ $= 109.8\ \mathrm{m}$
Specific behaviours
✓ writes integral for total distance ✓ correct distance

Question 18 **(7 marks)**

(a) Point A has coordinates $(-6, 1, 4)$ and plane Π has equation $2x + y - 2z = 17$. Determine

(i) a vector equation for the straight line through A perpendicular to Π.

(1 mark)

Solution
$\mathbf{r} = \begin{pmatrix} -6 \\ 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$
Specific behaviours
✓ correct equation

(ii) the perpendicular distance of A from Π. (3 marks)

Solution
$2(-6 + 2\lambda) + (1 + \lambda) - 2(4 - 2\lambda) = 17 \Rightarrow \lambda = 4$ $\mathbf{r} = \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix}, \quad \mathbf{r}_1 = \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix} - \begin{pmatrix} -6 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \\ -8 \end{pmatrix}$ $\lvert\mathbf{r}_1\rvert = 12$
Specific behaviours
✓ substitutes equation of line into equation of plane ✓ determines vector from point to plane ✓ calculates distance

(b) Prove that the perpendicular distance from the origin to the plane $\mathbf{r} \cdot \mathbf{n} = k$ is $\dfrac{k}{\lvert\mathbf{n}\rvert}$.

(3 marks)

Solution
Equation of line perpendicular to plane through origin is $\mathbf{r} = \lambda\mathbf{n}$. Line will intersect plane when $\lambda\mathbf{n} \cdot \mathbf{n} = k \Rightarrow \lambda = \frac{k}{\mathbf{n}\cdot\mathbf{n}} = \frac{k}{\lvert\mathbf{n}\rvert^2}$ Thus, closest point in plane to origin is $\mathbf{r} = \frac{k}{\lvert\mathbf{n}\rvert^2}\,\mathbf{n}$ Hence distance $d = \lvert\mathbf{r}\rvert = \frac{k}{\lvert\mathbf{n}\rvert^2}\lvert\,\mathbf{n}\,\rvert = \frac{k}{\lvert\mathbf{n}\rvert}$
Specific behaviours
✓ substitutes equation of line into equation of plane ✓ simplifies $\mathbf{n} \cdot \mathbf{n}$ and uses to obtain closest point to origin ✓ simplifies expression for distance

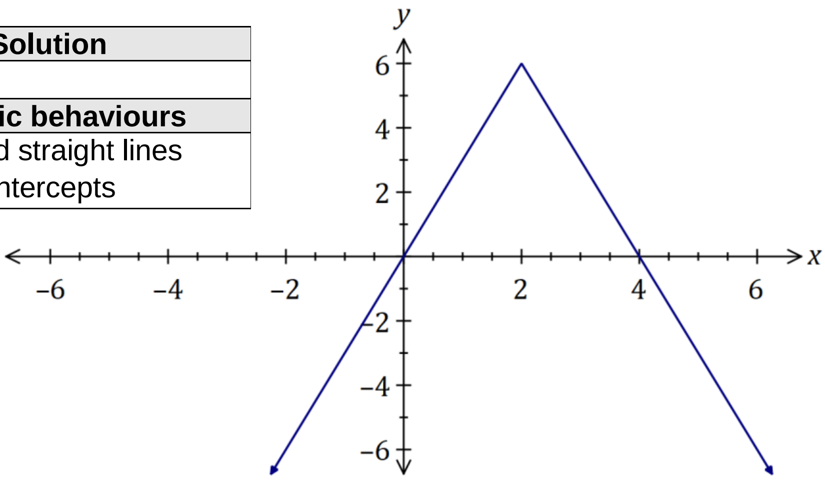
Alternative solution
An alternative proof could involve using $\lvert\mathbf{r}\rvert\lvert\mathbf{n}\rvert\cos\theta$ and explaining why $\theta = 0$.

Question 19 **(8 marks)**

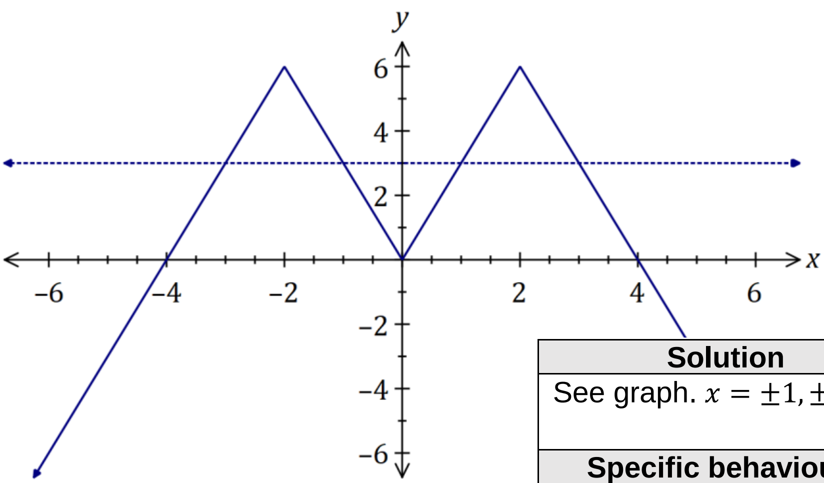
Let $f(x) = 6 - |3x - 6|$.

(a) Sketch the graph of $y = f(x)$ on the axes below. (2 marks)

Solution
See graph
Specific behaviours
✓ cusp and straight lines
✓ correct intercepts

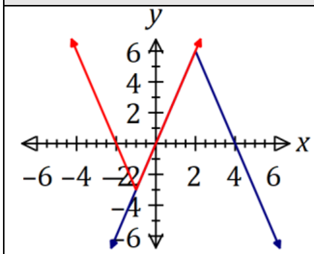


(b) Sketch the graph of $y = f(|x|)$ and hence solve $f(|x|) - 3 = 0$. (3 marks)



Solution
See graph. $x = \pm 1, \pm 3$
Specific behaviours
✓ required sketch
✓ adds $y = 3$
✓ solutions

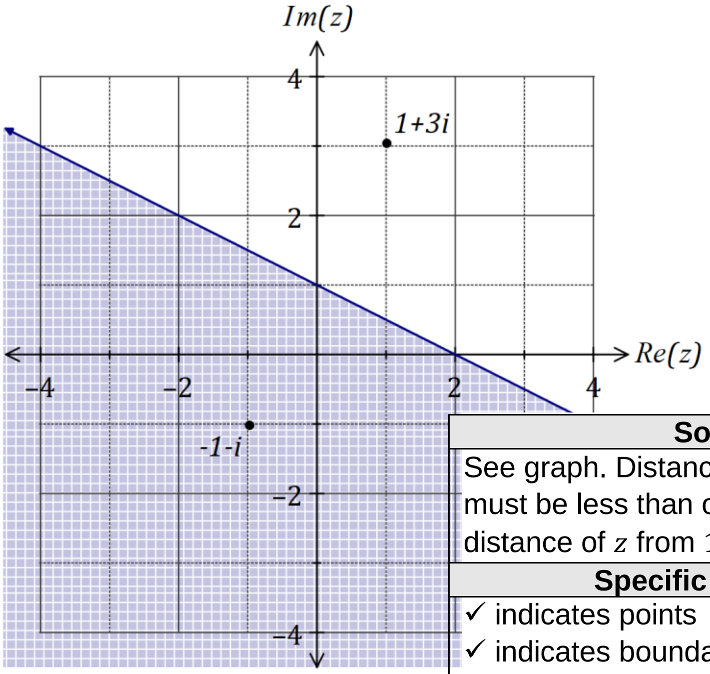
(c) Let $g(x) = a|x + b| + c$. The equation $f(x) = g(x)$ is true only for $-1 \le x \le 2$. Determine the value of each of the constants a, b and c. (3 marks)

Solution

$a = 3, \ b = 1, \ c = -3$
Specific behaviours
✓ sketch indicating situation
✓ two correct values
✓ all correct values

Question 20 **(6 marks)**

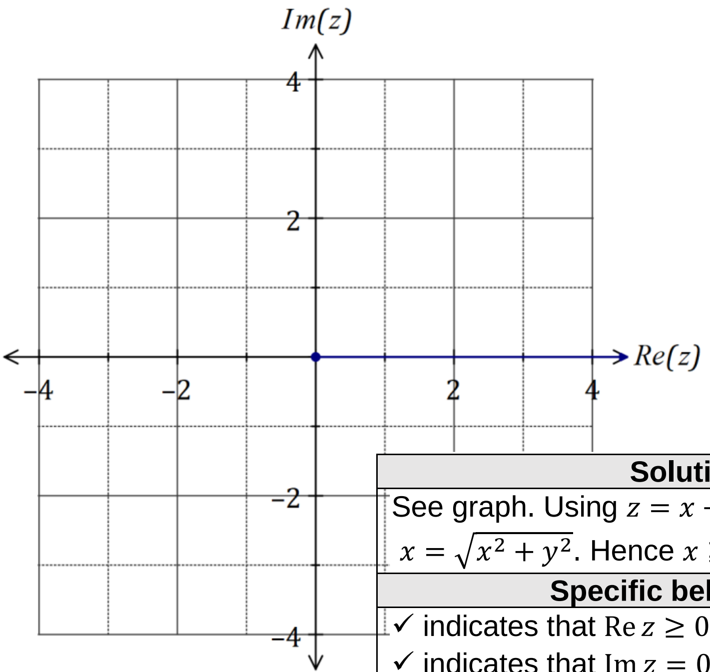
Sketch the locus of the complex number z given by

(a) $|z+1+i| \le |z-1-3i|$. (3 marks)



Solution
See graph. Distance of z from $-1-i$ must be less than or equal to the distance of z from $1+3i$
Specific behaviours
✓ indicates points ✓ indicates boundary ✓ indicates correct half plane

(b) $|z+3| = |z|+3$. (3 marks)



Solution
See graph. Using $z = x + iy$ leads to $x = \sqrt{x^2+y^2}$. Hence $x \ge 0$ and $y = 0$.
Specific behaviours
✓ indicates that $\text{Re}\, z \ge 0$ ✓ indicates that $\text{Im}\, z = 0$ ✓ indicates correct ray that includes origin

End of questions

Supplementary page

Question number: _________